

# Constraining quark mixing matrix in isosinglet vector-like down quark model from a fit to flavor-physics data

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We consider a model where the standard model is extended by the addition of a vector-like isosinglet down-type quark  $d'$ . We perform a  $\chi^2$  fit to the flavor physics data and obtain the preferred central values along with errors of all the elements of the measurable  $3 \times 4$  quark mixing matrix. We find that the data constrains  $|V_{tb}| \geq 0.99$  at  $3\sigma$ . Hence, no large deviation in  $|V_{tb}|$  is possible, even if the mixing matrix is allowed to be non-unitary. The fit also indicates that all the new-physics parameters are consistent with zero, and the mixing of the  $d'$  quark with the other three is constrained to be small.

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## I. INTRODUCTION

The standard model (SM) consists of three generations of quarks. In each generation, there are two quarks. However, there is no *a priori* reason for the number of quarks to be restricted to six. It may be possible to have heavier quarks whose effects have not been detected yet. The minimal extension of the SM in this direction can be obtained by adding a vectorlike isosinglet up-type or down-type quark to the SM particle spectrum [1–19]. Such exotic fermions can appear in  $E_6$  grand unified theories as well in models with large extra dimensions. Here we consider the extension of SM by adding a vector like down-type quark  $d'$ .

The ordinary  $Q_{em} = -1/3$  quarks mix with the  $d'$ . Because the  $d'_L$  has a different  $I_{3L}$  from  $d_L$ ,  $s_L$  and  $b_L$ ,  $Z$ -mediated flavour changing neutral current (ZFCNC) appear at the tree level in the left-handed sector. Thus the quark level transitions such as  $b \rightarrow s$ ,  $b \rightarrow d$ ,  $s \rightarrow d$  can occur at the tree level. The addition of a  $d'$  quark to the SM leads to a quark mixing matrix which is the  $3 \times 4$  upper submatrix of a  $4 \times 4$  quark-mixing matrix CKM4, which is an extension of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix in the SM. This model thus provides a self-consistent framework to study deviations of  $3 \times 3$  unitarity of the CKM matrix as well as flavor changing neutral currents at tree level.

Not all elements of the CKM matrix are measured directly; the values of the elements  $V_{tq}$  ( $q = d, s, b$ ) are determined from decays involving loops and using the unitarity of the  $3 \times 3$  CKM matrix. Hence one expects that due to the non unitarity of the quark mixing matrix in the ZFCNC model, sizable departure from the SM prediction may be possible. In this paper, we explore the possibility of such deviations. In order to make concrete predictions for the elements of the quark mixing matrix in ZFCNC model, we need to determine the independent parameters of the quark mixing matrix. The parametrization of  $4 \times 4$  unitary quark-mixing matrix requires six real parameters and three phases. Once we determine these nine parameters, we will be able to determine all the elements of the  $3 \times 4$  quark mixing matrix. Using these elements one can also determine the tree level ZFCNC couplings.

We use the Dighe-Kim (DK) parametrization [20–22] of the CKM4 matrix with

9 parameters, and perform a combined fit to these parameters using flavor-physics data. In addition to the direct measurements of the CKM4 matrix elements, the fit includes observables that have been well-measured and are consistent with the SM. These include: (i) the branching ratio of the inclusive decay  $\bar{B} \rightarrow X_s \mu^+ \mu^-$  in the high- $q^2$  and low- $q^2$  regions, (ii) the branching ratio of the exclusive decay  $\bar{B} \rightarrow \bar{K} \mu^+ \mu^-$  in the high- $q^2$  region, (iii) the branching ratios of  $\bar{B}_s \rightarrow \mu^+ \mu^-$  and  $\bar{B} \rightarrow X_s \nu \bar{\nu}$ , (iv) the mass differences in the  $B_d$  and  $B_s$  systems, (v) the measurement of the angle  $\gamma$  of the unitarity triangle from tree-level decays, (vi)  $\epsilon_K$  from  $K_L \rightarrow \pi\pi$ , (vii) the branching ratio of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \mu^+ \mu^-$ , and (viii)  $R_b$  and  $A_b$  from  $Z \rightarrow b\bar{b}$ .

The paper is organized as follows. In Sec. II, we define the model. In Sec. III, we present the details of the  $\chi^2$  fit. The results of the fit are presented in Sec. IV. We conclude in Sec. V with a discussion of the results.

## II. FCNC IN Z COUPLINGS

In SM the quark content is represented by:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R; \begin{pmatrix} c_L \\ s_L \end{pmatrix}, c_R, s_R; \begin{pmatrix} t_L \\ b_L \end{pmatrix}, t_R, b_R \quad (1)$$

Here left handed quarks are represented as doublet and right handed quarks are represented as singlet under  $SU(2)_L$ . Here we study a model where the quark sector is extended by the addition of an  $SU(2)$  siglet vector-like quark of charge  $(-1/3)$ , labelled  $d'$ . The mixing of this quark with the SM quarks of charge  $(-1/3)$  leads to a different structure for the CKM matrix. The  $3 \times 3$  mixing matrix connecting the charge  $(2/3)$  quarks to the charge  $(-1/3)$  quarks is no longer unitary, but is a submatrix of a  $4 \times 4$  unitary matrix. Without loss of generality, we choose to work in weak basis where the up quark mass matrix is diagonal and real. The down sector interaction eigenstates are then related to mass eigenstates by a  $4 \times 4$  unitary matrix. In this basis,  $4 \times 4$  unitary matrix appears for the bi-diagonalization of the  $4 \times 4$  mass matrix  $M$  of  $Q = -1/3$  quarks as

$$U^\dagger M M^\dagger U = \text{diag.}(m_d^2, m_s^2, m_b^2, m_{d'}^2) \quad (2)$$

In particular, the ZFCNC couplings of the down quarks are given by

$$\mathcal{L}_{FCNC}^Z = -\frac{e}{2 \sin \theta_w \cos \theta_w} U_{ij} \bar{d}_{iL} \gamma^\mu d_{jL} Z_\mu \quad (3)$$

where  $U_{ij} = -V_{4i}^* V_{4j}$  for  $i \neq j$  so the flavour changing neutral currents are given by  $U_{sd} = -V_{4s}^* V_{4d}$ ,  $U_{sb} = -V_{4s}^* V_{4b}$ ,  $U_{bd} = -V_{4b}^* V_{4d}$ .

$ V_{ud}  = (0.97425 \pm 0.00022)$	$\eta_c = 1.87 \pm 0.76[26]$
$ V_{us}  = (0.2252 \pm 0.0009)$	$\eta_t = 0.5765 \pm 0.0065[25]$
$ V_{cd}  = (0.230 \pm 0.011)$	$\eta_{ct} = 0.496 \pm 0.047[27]$
$ V_{cs}  = (1.006 \pm 0.023)$	$\Delta M_s = (17.72 \pm 0.04) \text{ ps}^{-1}[28]$
$ V_{ub}  = (0.00382 \pm 0.00021)$	$\Delta M_d = (0.507 \pm 0.004) \text{ ps}^{-1}[28]$
$ V_{cb}  = (40.9 \pm 1.0) \times 10^{-3}$	$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9} [29-31]$
$\gamma = (68.0 \pm 11.0)^\circ$	$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{low}} = (1.60 \pm 0.50) \times 10^{-6}[32][33]$
$A_b = 0.923 \pm 0.020[23]$	$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{high}} = (0.44 \pm 0.12) \times 10^{-6}[32][33]$
$R_b = 0.2158 \pm 0.0003[23]$	$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \nu) = (1.7 \pm 1.1) \times 10^{-10}$
$m_c/m_b = 0.29 \pm 0.02$	$\mathcal{B}(\bar{B} \rightarrow \bar{K} \mu^+ \mu^-)_{\text{high}} = (0.98 \pm 0.26) \times 10^{-7}[34]$
$m_t(m_t) = 163 \text{ GeV} [24]$	$\mathcal{B}(B \rightarrow X_c \ell \nu) = (10.61 \pm 0.17) \times 10^{-2}$
$B_K = 0.767 \pm 0.010[24]$	$\mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu_e) = (5.07 \pm 0.04) \times 10^{-2}$
$f_K = 0.1561 \pm 0.0011 [24]$	$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-) < 2.5 \times 10^{-9} [35]$
$\Delta M_K = (0.5292 \pm 0.0009) \times 10^{-2} \text{ ps}^{-1}$	$\mathcal{B}(K^+ \rightarrow \mu^+ \nu_\mu) = 0.06355 \pm 0.0011$
$f_{B_d} \sqrt{B_{B_d}} = (0.226 \pm 0.013) \text{ GeV}[24]$	$ \epsilon_k  \times 10^3 = 2.228 \pm 0.011$
$f_{B_s} \sqrt{B_{B_s}} = (0.279 \pm 0.013) \text{ GeV} [24]$	$\tau_{K_L} = (5.116 \pm 0.021) \times 10^{-8} \text{ s}$
$\eta_B = 0.55 \pm 0.01[25]$	$\tau_{K^+} = (1.238 \pm 0.0021) \times 10^{-8} \text{ s}$

TABLE I: Inputs used to constrain the parameter space. When not explicitly stated, we take the inputs from the Particle Data Group [36]. For  $V_{ub}$ , we use the weighted average from the inclusive and exclusive semileptonic decays,  $V_{ub}^{inc} = (44.1 \pm 3.1) \times 10^{-4}$  and  $V_{ub}^{exc} = (32.3 \pm 3.1) \times 10^{-4}$ .

### III. THE $\chi^2$ FUNCTION FOR THE FIT

For the fit, we define the total  $\chi^2$  function as

$$\begin{aligned} \chi_{\text{total}}^2 = & \chi_{\text{CKM}}^2 + \chi_{\gamma}^2 + \chi_{B \rightarrow X_s \mu^+ \mu^- : \text{low}}^2 + \chi_{B \rightarrow X_s \mu^+ \mu^- : \text{high}}^2 + \chi_{B \rightarrow K \mu^+ \mu^- : \text{high}}^2 \\ & + \chi_{B \rightarrow X_s \nu \bar{\nu}}^2 + \chi_{B_s \rightarrow \mu^+ \mu^-}^2 + \chi_{B_s}^2 + \chi_{B_d}^2 + \chi_{|\epsilon_K|}^2 + \chi_{K^+ \rightarrow \pi^+ \nu \bar{\nu}}^2 \\ & + \chi_{K_L \rightarrow \mu^+ \mu^-}^2 + \chi_{R_b}^2 + \chi_{A_b}^2. \end{aligned} \quad (4)$$

In our analysis  $\chi^2$  of an observable  $A$  is defined as

$$\chi_A^2 = \left( \frac{A - A_{\text{exp}}^c}{A_{\text{exp}}^{\text{err}}} \right)^2, \quad (5)$$

where the measured value of  $A$  is  $(A_{\text{exp}}^c \pm A_{\text{exp}}^{\text{err}})$ . The individual components of the function  $\chi_{\text{total}}^2$ , i.e the  $\chi^2$  of different observables that we are using as inputs, are defined in the following subsections.

#### A. Direct measurements of the CKM elements

The contribution to the  $\chi^2$  from the direct measurements of the magnitudes of the CKM elements is given by

$$\begin{aligned} \chi_{\text{CKM}}^2 = & \left( \frac{|V_{us}| - 0.2252}{0.0009} \right)^2 + \left( \frac{|V_{ud}| - 0.97425}{0.00022} \right)^2 + \left( \frac{|V_{cs}| - 1.006}{0.023} \right)^2 \\ & + \left( \frac{|V_{cd}| - 0.230}{0.011} \right)^2 + \left( \frac{|V_{ub}| - 0.00382}{0.00021} \right)^2 + \left( \frac{|V_{cb}| - 0.0409}{0.001} \right)^2. \end{aligned} \quad (6)$$

#### B. CKM angle $\gamma$

1. For the CKM angle  $\gamma$ , we have

$$\chi_{\gamma}^2 = \left( \frac{\delta_{ub} - 68 (\pi/180)}{11 (\pi/180)} \right)^2, \quad (7)$$

#### C. Branching ratio of $\bar{B} \rightarrow X_s \mu^+ \mu^-$

The effective Hamiltonian for the quark-level transition  $b \rightarrow s \mu^+ \mu^-$  in the SM can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (8)$$

where the form of the operators  $O_i$  and the expressions for calculating the coefficients  $C_i$  are given in Ref. [37]. The  $Z\bar{b}s$  coupling generated in the ZFCNC model changes the values of the Wilson coefficients  $C_{9,10}$ . The Wilson coefficients  $C_{9,10}^{\text{tot}}$  in the ZFCNC model can be written as [19]

$$C_9^{\text{tot}} = C_9^{\text{eff}} - \frac{\pi}{\alpha} \frac{U_{sb}}{V_{ts}^* V_{tb}} (4 \sin^2 \theta_W - 1) \quad (9)$$

$$C_{10}^{\text{tot}} = C_{10} - \frac{\pi}{\alpha} \frac{U_{sb}}{V_{ts}^* V_{tb}}. \quad (10)$$

We use the SM Wilson coefficients as given in Ref. [38].

The theoretical prediction for the branching fraction of  $\bar{B} \rightarrow X_s \mu^+ \mu^-$  in the intermediate  $q^2$  region ( $7 \text{ GeV}^2 \leq q^2 \leq 12 \text{ GeV}^2$ ) is rather uncertain due to the nearby charmed resonances. The predictions are relatively cleaner in the low- $q^2$  ( $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ ) and the high- $q^2$  ( $14.4 \text{ GeV}^2 \leq q^2 \leq m_b^2$ ) regions. We therefore consider both low- $q^2$  high- $q^2$  region in the fit.

The quantity we use for  $\bar{B} \rightarrow X_s \mu^+ \mu^-$  is

$$D = \mathcal{B}(\bar{B} \rightarrow X_s \mu^+ \mu^-) \frac{4\pi^2 f(\hat{m}_c) \kappa(\hat{m}_c)}{\alpha^2 \mathcal{B}(B \rightarrow X_c e \bar{\nu})}. \quad (11)$$

We use the value of  $\hat{m}_c$  given in Table I and obtain

$$f(\hat{m}_c) = 0.542 \pm 0.045, \quad \kappa(\hat{m}_c) = 0.88 \pm 0.003. \quad (12)$$

With all the other inputs given in Table I, and adding an overall corrections about 30% to the  $\mathcal{B}(\bar{B} \rightarrow X_s \mu^+ \mu^-)_{\text{high}}$ , which includes the non-perturbative corrections, we estimate

$$D_{\text{low,exp}} = 5.3295 \pm 1.7255, \quad D_{\text{high,exp}} = 1.4656 \pm 0.4185. \quad (13)$$

The contribution of this quantity to  $\chi^2$  is then

$$\chi_{B \rightarrow X_s \mu^+ \mu^- : \text{low}}^2 = \left( \frac{D_{\text{low}} - 5.3295}{1.7255} \right)^2, \quad \chi_{B \rightarrow X_s \mu^+ \mu^- : \text{high}}^2 = \left( \frac{D_{\text{high}} - 1.4656}{0.4185} \right)^2. \quad (14)$$

Using the expression for  $\bar{B} \rightarrow X_s \mu^+ \mu^-$  given in [19], one can easily find an expression for  $D$ 's.

### D. Branching ratio of $\bar{B} \rightarrow \bar{K} \mu^+ \mu^-$

In order to include  $\bar{B} \rightarrow \bar{K} \mu^+ \mu^-$ , we define

$$B = \frac{3\mathcal{B}(\bar{B} \rightarrow \bar{K} \mu^+ \mu^-) 2^9 \pi^5 m_B^3}{G_F^2 \alpha^2 \tau_B} \quad (15)$$

Using the expression for  $\bar{B} \rightarrow \bar{K} \mu^+ \mu^-$  given in [39], one can easily find a theoretical expression for  $B$ . With all the other inputs given in Table I, and adding an overall corrections about 60% to the  $\mathcal{B}(\bar{B} \rightarrow \bar{K} \mu^+ \mu^-)_{16.00 \text{ GeV}^2 \leq q^2 \leq 22.86 \text{ GeV}^2}$ , which includes uncertainties due to form factors & CKM matrix elements and subleading  $1/m_b$ -corrections to the  $\bar{B} \rightarrow \bar{K} \mu^+ \mu^-$  matrix elements, we estimate

$$B = 404.935 \pm 273.08. \quad (16)$$

The contribution to  $\chi^2$  from  $\mathcal{B}(\bar{B} \rightarrow \bar{K} \mu^+ \mu^-)$  is then

$$\chi_{\bar{B} \rightarrow \bar{K} \mu^+ \mu^-}^2 = \left( \frac{B - 404.935}{273.08} \right)^2. \quad (17)$$

### E. Branching ratio of $\bar{B}_s \rightarrow \mu^+ \mu^-$

In order to include  $\mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-)$ , we define

$$B_{\text{lep}} = \frac{16\pi^3 \mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-)}{G_F^2 \alpha^2 M_{B_s} m_\mu^2 f_{B_s}^2 \tau_{B_s} \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}}}. \quad (18)$$

Using the inputs given in Table I, we obtain

$$B_{\text{lep,exp}} = 0.0290 \pm 0.0080. \quad (19)$$

The contribution to  $\chi^2$  from  $\mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-)$  is then

$$\chi_{\bar{B}_s \rightarrow \mu^+ \mu^-}^2 = \left( \frac{B_{\text{lep}} - 0.0290}{0.0080} \right)^2, \quad (20)$$

Using the expression for  $\mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-)$  given in [19], one can easily determine theoretical expression for  $B_{\text{lep}}$ .

### F. Branching ratio of $\bar{B} \rightarrow X_s \nu \bar{\nu}$

In order to include the contribution to  $\chi^2$  from  $\mathcal{B}(\bar{B} \rightarrow X_s \nu \bar{\nu})$ , we define

$$B_{\text{inv}} = \frac{\mathcal{B}(\bar{B} \rightarrow X_s \nu \bar{\nu})}{\mathcal{B}(B \rightarrow X_c e \bar{\nu})} \frac{2\pi^2 \sin^4 \theta_W f(\hat{m}_c) \kappa(\hat{m}_c)}{\bar{\eta} \alpha^2}. \quad (21)$$

Using the inputs given in Table I, we obtain

$$B_{\text{inv,exp}} = 0.0 \pm 42.8889. \quad (22)$$

Thus the contribution of  $\mathcal{B}(\bar{B} \rightarrow X_s \nu \bar{\nu})$  to  $\chi^2$  is

$$\chi_{\bar{B} \rightarrow X_s \nu \bar{\nu}}^2 = \left( \frac{B_{\text{inv}} - 0.0}{42.8889} \right)^2. \quad (23)$$

Theoretical expression for  $B_{\text{inv}}$  can easily be determined using the expression for  $\mathcal{B}(\bar{B} \rightarrow X_s \nu \bar{\nu})$  given in [19].

### G. $B_q$ - $\bar{B}_q$ mixing ( $q = d, s$ )

To calculate  $\chi_{B_q}^2$  ( $q = d, s$ ) for  $B_q$ - $\bar{B}_q$  mixing, we use the quantities

$$B_{\text{mix}}^q = \frac{6\pi^2 \Delta M_q}{G_F^2 M_W^2 M_{B_q} \eta_B \hat{B}_{bq} f_{B_q}^2 |E(x_t)|}. \quad (24)$$

With the inputs given in Table I, we get

$$\begin{aligned} B_{\text{mix,exp}}^s &= (1.48 \pm 0.14) \times 10^{-3}, \\ B_{\text{mix,exp}}^d &= (6.45 \pm 0.75) \times 10^{-5}. \end{aligned} \quad (25)$$

Then one gets

$$\chi_{B_s}^2 = \left( \frac{B_{\text{mix}}^s - 1.48 \times 10^{-3}}{0.14 \times 10^{-3}} \right)^2, \quad (26)$$

$$\chi_{B_d}^2 = \left( \frac{B_{\text{mix}}^d - 6.45 \times 10^{-5}}{0.75 \times 10^{-5}} \right)^2. \quad (27)$$

### H. Indirect CP violation $\epsilon_K$ in $K_L \rightarrow \pi\pi$

To calculate the contribution to  $\chi^2$  from  $|\epsilon_K|$ , we use the quantity

$$K_{\text{mix}} = \frac{12\sqrt{2}\pi^2 (\Delta M_K)_{\text{exp}} |\epsilon_K|}{G_F^2 M_W^2 f_K^2 m_K \hat{B}_K k_\epsilon} \quad (28)$$



With the theoretical and experimental inputs given in Table I, we find

$$K_{\text{mix, exp}} = (1.69 \pm 0.05) \times 10^{-7} . \quad (29)$$

The contribution to  $\chi^2$  from  $|\epsilon_K|$  is then

$$\chi_{|\epsilon_K|}^2 = \left( \frac{K_{\text{mix}} - 1.69 \times 10^{-7}}{0.05 \times 10^{-7}} \right)^2 + \chi_{\eta_{ct}}^2 , \quad (30)$$

where

$$\chi_{\eta_{ct}}^2 = \left( \frac{\eta_{ct} - 0.496}{0.047} \right)^2 . \quad (31)$$

Using the expression for  $|\epsilon_K|$  given in [13], it is straightforward to find an expression for  $K_{\text{mix}}$ . In order to take into account the error in  $\eta_{ct}$  which appears in the theoretical expression of  $|\epsilon_K|$ , we consider it to be a parameter and have added a term,  $\chi_{\eta_{ct}}^2$ , in  $\chi^2$ .

### I. Branching fraction of the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Unlike other K decays,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is dominated by the short-distance interactions. The long-distance (LD) contribution to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is about 3 orders of magnitude smaller than that of the short-distance (SD) [40, 41].

In order to include  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ , we define

$$\chi_{K^+ \rightarrow \pi^+ \nu \bar{\nu}}^2 = \left( \frac{K_{\text{slep}} - 7.37 \times 10^{-5}}{4.77 \times 10^{-5}} \right)^2 + \chi_X^2 , \quad (32)$$

where

$$\chi_X^2 = \left( \frac{X_e^{nl} - 10.6 \times 10^{-4}}{1.5 \times 10^{-4}} \right)^2 + \left( \frac{X_\tau^{nl} - 7.1 \times 10^{-4}}{1.4 \times 10^{-4}} \right)^2 , \quad (33)$$

Using the input Table I, we obtain

$$K_{\text{slep}} = \frac{2\pi^2 \sin^4 \theta_w Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\alpha^2 r_K Br(K^+ \rightarrow \pi^0 e^+ \nu)} = (7.37 \pm 4.77) \times 10^{-5} . \quad (34)$$

Using the expression for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  given in [13], it is straightforward to find an expression for  $K_{\text{slep}}$ . In order to include the error in quantities  $X_e^{nl}$  and  $X_\tau^{nl}$  which appears in the theoretical expression of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ , we consider them to be a parameter and have added a term,  $\chi_X^2$ , in  $\chi^2$ .

### J. Branching fraction of the decay $K_L \rightarrow \mu^+ \mu^-$

Unlike  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \mu^+ \mu^-$  is not dominated by clean SD effects the LD and SD contributions are comparable in size. In order to extract bounds on the SD contribution to the branching ratio of  $K_L \rightarrow \mu^+ \mu^-$ , it is extremely important to have a theoretical control on the  $K_L \rightarrow \gamma \gamma$  form factors with off-shell photons. A conservative bound of  $2.5 \times 10^9$  on  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$  SD was obtained in Ref. [35]. We use this bound to constrain the CKM4 parameters. In order to include  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$ , we define

$$\chi_{K_L \rightarrow \mu^+ \mu^-}^2 = \left( \frac{K_{\text{lep}} - 0.0}{5.89 \times 10^{-6}} \right)^2 + \chi_{Y^{NL}}^2, \quad (35)$$

where

$$\chi_{Y^{NL}}^2 = \left( \frac{Y^{NL} - 2.94 \times 10^{-4}}{0.28 \times 10^{-4}} \right)^2, \quad (36)$$

Using the input Table I, we obtain

$$K_{\text{lep}} = \frac{\pi^2 \sin^4 \theta_w Br(K_L \rightarrow \mu^+ \mu^-) \tau_{K^+}}{\alpha^2 Br(K^+ \rightarrow \mu^+ \nu) \tau_{K_L}} = (0.0 \pm 5.89) \times 10^{-6}. \quad (37)$$

Using the expression for  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$  given in [13], the theoretical expression for  $K_{\text{lep}}$  can be easily obtained. The quantity  $Y^{NL}$  appears in the theoretical expression for  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$ . In order to include error in  $Y_{NL}$ , we consider it to be a parameter and have added a term,  $\chi_{Y^{NL}}^2$ , in  $\chi^2$ .

### K. $Z \rightarrow b \bar{b}$ decay

The contribution from the forward-backward asymmetry in  $Z \rightarrow b \bar{b}$  is

$$\chi_{A_b}^2 = \left( \frac{A_b - 0.923}{0.020} \right)^2, \quad (38)$$

The weak isovector part of  $Z \rightarrow b \bar{b}$  is reduced by  $(1 - |V_{4b}|^2)$  so the  $\chi^2$  contribution from  $R_b$  is

$$\chi_{R_b}^2 = \left( \frac{R_b - 0.2164}{0.00073} \right)^2, \quad (39)$$

where expression for  $R_b$  and  $A_b$  can be seen in [14].

#### IV. RESULTS OF THE FIT

The results of these fits are presented in Table II. It is obvious that the  $\chi^2$  per degree of freedom is small in each case, indicating that all the fits are good.

Parameter	SM	ZFCNC
$\lambda$	$0.2260 \pm 0.0006$	$0.2260 \pm 0.0006$
$A$	$0.7744 \pm 0.0171$	$0.7787 \pm 0.0172$
$C$	$0.4147 \pm 0.0241$	$0.4245 \pm 0.0250$
$\delta_{ub}$	$1.0369 \pm 0.0869$	$1.1397 \pm 0.1469$
$p$	—	$0.8753 \pm 1.0308$
$q$	—	$0.0354 \pm 0.0413$
$r$	—	$0.0530 \pm 0.2808$
$\delta_{ud'}$	—	$0.7271 \pm 1.3332$
$\delta_{cd'}$	—	$0.0071 \pm 5.4917$
$\chi^2/d.o.f.$	9.30/16	5.10/9

TABLE II: The results of the fit to the parameters of CKM and ZFCNC.

The results of the fit for the SM is consistent with that obtained in Ref. [36]. In the SM fit, apart from the constraints mentioned in the previous section, we have also included the  $3 \times 3$  unitarity constraints. The best fit values of the parameters of the SM CKM matrix are not affected much by the addition of a vector-like isosinglet down-type quark  $d'$ . On the other hand, the new real parameters  $p$ ,  $q$ ,  $r$  are consistent with zero, which is not surprising since the SM fit is a good one. This also is consistent with the observation that no meaningful constraints are obtained on the new phases  $\delta_{ud'}$  and  $\delta_{cd'}$ : since vanishing  $p$ ,  $q$  imply vanishing  $V_{ud'}$ ,  $V_{cd'}$ , respectively, the phases of these two elements have no significance.

Qunatity	SM	ZFCNC
$ V_{ud} $	$0.9745 \pm 0.0001$	$0.9745 \pm 0.0001$
$ V_{us} $	$0.2260 \pm 0.0006$	$0.2260 \pm 0.0006$
$ V_{ub} $	$(3.71 \pm 0.23) \times 10^{-3}$	$(3.8162 \pm 0.2420) \times 10^{-3}$
$ V_{ud'} $	—	$0.0101 \pm 0.0119$
$ V_{cd} $	$0.2260 \pm 0.0006$	$0.2260 \pm 0.0006$
$ V_{cs} $	$0.9745 \pm 0.0001$	$0.9745 \pm 0.0001$
$ V_{cb} $	$0.0396 \pm 0.0008$	$0.0398 \pm 0.0009$
$ V_{cd'} $	—	$0.0018 \pm 0.0021$
$ V_{td} $	$0.0078 \pm 0.0004$	$0.0081 \pm 0.0006$
$ V_{ts} $	$0.0390 \pm 0.0008$	$0.0391 \pm 0.0009$
$ V_{tb} $	1	$0.9999 \pm 0.0007$
$ V_{td'} $	—	$0.0120 \pm 0.0635$

TABLE III: Magnitudes of the  $3 \times 4$  CKM elements obtained from the fit.

The magnitude of elements of the  $3 \times 4$  quark mixing matrix, obtained by using the fit values presented in Table II, are given in Table III. Clearly all new elements of the quark mixing matrix are constrained to be small and consistent with zero. The allowed values of ZFCNC couplings  $U_{sd}$ ,  $U_{sb}$  and  $U_{db}$  are given in Table IV.

Quantity	ZFCNC
$U_{sd}$	$(0.1947 \pm 1.6524) \times 10^{-4}$
$\text{Arg}U_{sd}$	$(1.1786 \pm 3.4482)$
$U_{sb}$	$(0.1534 \pm 0.9986) \times 10^{-4}$
$\text{Arg}U_{sb}$	$(1.9398 \pm 3.5580)$
$U_{db}$	$(0.1127 \pm 0.6065) \times 10^{-3}$
$\text{Arg}U_{db}$	$(2.3803 \pm 1.4045)$

TABLE IV: Magnitude and phase of ZFCNC couplings

## V. CONCLUSIONS

In this paper we consider the minimal extension of SM by addition of an isosinglet, vector like down-type quark  $d'$ . Using input from many flavor-physics processes, we perform a  $\chi^2$  fit to constrain the elements of the  $3 \times 4$  quark-mixing matrix and the ZFCNC couplings. The fit takes into account both experimental errors and theoretical uncertainties.

We conclude the following:

- The best-fit values of all three new real parameters of the CKM4 matrix are consistent with zero. This is expected since the fit to the SM is also excellent.
- Due to the non unitarity of the quark mixing matrix, one can expect deviation of  $|V_{tb}|$  from unity in this model. In the SM,  $|V_{tb}|$  is determined using the unitarity condition. The direct determination of  $|V_{tb}|$  without assuming unitarity is possible from the single top-quark-production cross section. The CDF and D0 measurement gives  $|V_{tb}| = 0.87 \pm 0.07$  [42–44] whereas the CMS measurements gives  $|V_{tb}| = 1.14 \pm 0.22$  [45]. Although the present measurements have large errors, they do not rule out large deviations of  $|V_{tb}|$  from unity. We find  $|V_{tb}| = 0.9999 \pm 0.0007$ . Thus, at  $3\sigma$ , we have  $|V_{tb}| \geq 0.99$ . Therefore this model cannot account for any large deviation of  $|V_{tb}|$  from unity.
- The values of  $V_{ts}$  and  $V_{tb}$  in this model are close to their SM predictions.
- The mixing of the  $d'$  quark with the other three is constrained to be  $|V_{ud'}| < 0.05$ ,  $|V_{cd'}| < 0.01$ , and  $|V_{td'}| < 0.21$  at  $3\sigma$ .
- The tree level ZFCNC couplings are constrained to be small. At  $3\sigma$ ,  $U_{sd} < 5.16 \times 10^{-4}$ ,  $U_{sb} < 3.16 \times 10^{-4}$  and  $U_{db} < 1.95 \times 10^{-3}$ .

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